

Further Risp 2: The Spiral-Line Area

Level 1

Using a graphing program, draw the polar curve $r = n\theta$, where n is a positive real number. What happens as n increases and decreases?

Now add to the graph the line $\theta = 1 + \frac{1}{n}$. What happens as n increases and decreases now?

Level 2

Can you find the area A enclosed by the curve above and the line above?

Does A have a stationary point as n varies? If so, for what value of n does this happen, and what is the nature of the point?

Can you generalise this result to the line $\theta = a + \frac{b}{n}$, where $a, b > 0$?

Level 3

Are there any other decreasing functions of n , say $f(n)$, where working with the boundary line $\theta = f(n)$ gives A a stationary point for some value of n ?

What happens if you choose $\theta = e^{-n}$?

Level 4

Suppose the boundary line is $\theta = f(n)$, where f is such that A remains constant as n increases. Can you find $f(n)$?