An Introduction to Elliptic Curves

Remember the story about Hardy and Ramanujan
and the number 1729?

Can you write 1729 as the sum of two cubes
in two different ways?

Can you sketch the curve $x^3 + y^3 = 1729$?

How many integer points can you mark in on this curve?
(That's points with integer coordinates.)

Suppose we decide to pick two of these points
and join them with a straight line. What can happen?

Pick a pair of these points so that the line joining them
cuts the curve again in a third point.
Can you find the coordinates of this third point?
(You don't need to multiply everything out!)

Why must this third point be a rational point
(that's a point with rational coordinates)
given our starting points?

Show that a straight line joining any two rational points
must cut the curve again in another rational point.

Can we say we have 'added' two rational points
on the curve to get another here?
Could the set of rational points on the curve
together with this addition operation form a group?

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