The Queens Problem

In Elizabeth II's Diamond Jubilee year, why not pose our young mathematicians a royal question to mark the event? How about this;

Given a 60 by 60 chessboard,
how many queens can you place upon it so that no queen can be taken by any other?

The answer cannot be more than 60, since the Pigeon-hole Principle says that if you post n + 1 letters into n pigeonholes, there must be at least two in one pigeonhole. If we therefore have 61 or more queens, there must be at least two queens in some row, which would mean they could take each other. So we have an upper bound for our answer of 60, and this upper bound can in fact be reached – it's possible to place 60 queens onto the board so that no queen threatens any other. (In general, given an n by n board, you can always place n queens upon it so that no one can take any other, as long as n is 4 or bigger). Taking the more familiar case of an 8 by 8 board, one possible arrangement is:

		Q					
					Q		
	Q						
						Q	
Q							
			Q				
							Q
				Q			

A really tough question now; in how many fundamentally different ways (calling rotations and reflections the same) can this be done? This is a sequence that gets very big extremely quickly.

n	4	5	6	7	8	9	10	11	12	13	14
number of unique ways	1	2	1	6	12	46	92	341	1787	9233	45752

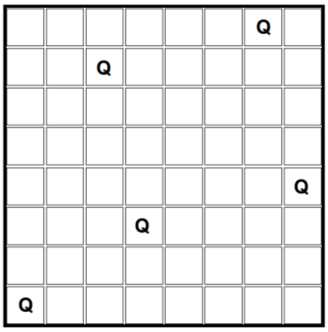
Strangely the case for n = 6 has fewer solutions than for n = 5 (our students might be glad to know there is no known formula for this sequence). Thankfully, there are some far easier questions that are accessible to everyone. What is the maximum number of squares that a queen can threaten on an 8 by 8 board? The minimum? Our students may have wondered what the ${}^{n}C_{r}$ button means on their calculators – now a good time to enlighten them. How many ways can you put 8 queens onto an 8 by 8 board? The answer is '64 choose 8' or ${}^{64}C_{8}$ (counting rotations and reflections as different), which is 4 426 165 368. Of these, just 92 are solutions to the problem. So if we place 8 queens at random onto the board, what is the probability that we hit on a solution? The chance is about 1 in 50 million – you have a better chance of matching all six numbers for the Lottery.

The Queens Problem represents a neat challenge to the computer programmer. With a little ingenuity, it is possible to find a surprisingly small amount of code that will find a solution, at least if time does not beat you! The University of Utah have a delightful applet at

http://www.math.utah.edu/~alfeld/queens/queens.html

that shows you how you and your computer might set about finding a solution for any n.

Finally, a related question – on an n by n board, what is the minimum number of queens needed so that every square is either occupied or threatened? It is best to start here with smaller boards; for the 8 by 8 board, the answer is 5.



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