MSV 5: Answers

An exercise designed to ensure that students
know how to find E(X) and Var(X) for a discrete random variable,
and that the probabilities in a probability distribution
will always add to one.

It is a help in comparing the fractions
to put them over a common denominator.

So the fractions (1/25, 2/15, 1/5, 3/5, 2/3, 19/25) become
(3/75, 10/75, 15/75, 45/75, 50/75, 57/75).

We need triples from the bag that will add to 1.
The only possibilities are (1/25, 1/5, 19/25), and (2/15, 1/5, 2/3).

These generate six distributions each, giving twelve possibilities in all.
To find the distributions with the same mean and variance, we have

x	a	b	c
P(X=x)	$\frac{19}{25}$	$\frac{1}{5}$	$\frac{1}{25}$

x	d	e	f
P(X=x)	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{2}{15}$

Knowing the mean is **209/375** in both cases gives

285a + 75b + 15c = 209, and **250d + 75e + 50f = 209**.

Now we can simply check the possibilities. For the right-hand equation,
250d is 190 or 150 or 10, 75e is 3 or 45 or 57, and 50f is 2 or 30 or 38.
The only combination that will add to 209 is 150 + 57 + 2.
We can use similar logic for the left-hand equation.

So the answers are (with shared variance **6344/140625**):

x	$\frac{2}{3}$	$\frac{2}{15}$	$\frac{3}{5}$
P(X=x)	$\frac{19}{25}$	$\frac{1}{5}$	$\frac{1}{25}$

x	$\frac{3}{5}$	$\frac{19}{25}$	$\frac{1}{25}$
P(X=x)	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{2}{15}$

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