From the editor

2014 and all that

No doubt the first thing you do on the first day of each year is to factorise the number of that year into its prime factors. Thus

$$2014 = 2 \times 19 \times 53.$$

Unfortunately you have been beaten to it by the English mathematician John Pell (16-10-1685), who factorised the first

100 000 numbers into their prime factors, a labour of love if ever there was one. John Pell's name is known today as attached to equations of the form $x^2 - dy^2 = 1$, where d is a perfect integer which is not a perfect square. These are to be solved in integers. There is no evidence that he ever considered such equations. They were wrongly attributed to him by Euler, and should more correctly be known as Fermat equations. But no matter.

One of our frequent contributors, Jonny Griffiths, has gone much further with the year. He defines 2014_n as n lots of 2014 concatenated, so that $2014_3 = 201420142014$. He asks whether any of these numbers has a prime factor occurring more than once. Or, to put it another way, is any of them divisible by a perfect square bigger than 1? Certainly some are – for example,

$$2014_9 = 2014(1+10^4+10^8+...10^{32})$$

$$= 2014(1^0+1^4+1^8...+1^{32})(\text{mod } 9) = 0 \ (\text{mod}9).$$

So that 3^2 divides 2014_9, and , by the same token, 2014_{9k} is divisible by 3^2. Jonny has found that 2014_n is not divisible by a square bigger than 1 for n = 1 to 8.

Clearly the number 2014 is a bit of a red herring – the crucial things are the numbers

$$10^0+10^4+10^8+...10^{4n}.$$

Jonny has not found any of these to be prime, although they seem reluctant to be divisible by perfect squares. But

$$10^0+10^4+10^8...+10^{80}$$

is divisible by 7^2. This can be seen by judicious use of modular arithmetic and a calculator. Thus

$$10^0+10^4+10^8...+10^{80}= 100^0+100^2 +100^4...+100^{40}$$

$$= 2^0+2^2+2^4+2^8...2^{40}(\mathrm{mod}49)$$

$$= (2^{42}-1)/3 \ \ (\mathrm{mod}49) \text{ and}$$

$$2^{42}- 1 = 64^7-1 = 15^7-1 = 0 \ (\mathrm{mod}49)$$

Jonny gives the horrific factorisation

3·7^2·13·29·37·43·127·239·281·1933·2689·4649·9901·226549·459691·909091·10838689·121499449·4458192223320340849

for this number. He suggests readers might find it fun to tackle these numbers with Wolfram Alpha.

On a lighter note, perhaps it is time for a partial resurrection of our annual puzzle. The idea is to see how far you can get expressing numbers using the digits of the year once and once only in order, using the operations +, - x, divide, factorial, square root, and concatenation. Thus, to get you started, 1 = - 2 + 0 – 1 + 4.

From the editor (Jonny's version)

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No doubt the first thing you do on the first day of each year is to factorise the number of that year into its prime factors. Thus

$$2014 = 2 \times 19 \times 53.$$

Unfortunately you have been beaten to it by the English mathematician John Pell (16-10-1685), who factorised the first

100 000 numbers into their prime factors, a labour of love if ever there was one. John Pell's name is known today as attached to equations of the form $x^2 - dy^2 = 1$, where d is a perfect integer which is not a perfect square. These are to be solved in integers. There is no evidence that he ever considered such equations. They were wrongly attributed to him by Euler, and should more correctly be known as Fermat equations. But no matter.

One of our frequent contributors, Jonny Griffiths, has gone further with the current year. Trying to find a quick numerical problem to set his students, he wrote down as his first few numbers 2014, 20142014, 201420142014... When he tried factorising these, they seemed to be remarkably 'reluctant' to have square factors. So he came up with a conjecture: 'if we define 2014_n as n lots of 2014 concatenated, none of them divisible by a perfect square bigger than 1'. The conjecture holds for n = 1 to 8 – but then

$$2014_9 = 2014(1+10^4+10^8+...10^{32})$$

$$= 2014(1^0+1^4+1^8...+1^{32})(\text{mod } 9) = 0\ (\text{mod}9).$$

So that 3^2 divides 2014_9, for obvious reasons when we reflect upon it, and, by the same token, 2014_{9k} is divisible by 3^2.

Clearly the number 2014 is something of a red herring here – the crucial things are the numbers

$$10^0 + 10^4 + 10^8 + ... + 10^{4n}.$$

Jonny has not yet found any of these to be prime, although there seems to be no obvious reason why they are guaranteed to be composite.

Jonny's conjecture breaks down for square factors other than 9. The number

$$10^0 + 10^4 + 10^8 ... + 10^{80}$$

is divisible by 7^2. This can be seen by judicious use of modular arithmetic and a calculator. Thus

$$10^0+10^4+10^8...+10^{80}= 100^0+100^2 +100^4...+100^{40}$$

$$= 2^0+2^2+2^4+2^8...2^{40} \text{ (mod49)}$$

$$= (2^{42}-1)/3 \text{ (mod49) and}$$

$$2^{42} - 1 = 64^7 - 1 = 15^7 - 1 = 0 \text{ (mod49)}$$

The number 2013_{21} factorises as

$3{\cdot}7^2{\cdot}13{\cdot}29{\cdot}37{\cdot}43{\cdot}127{\cdot}239{\cdot}281{\cdot}1933{\cdot}2689{\cdot}4649{\cdot}9901{\cdot}226549{\cdot}459691{\cdot}909091{\cdot}10838689{\cdot}121499449{\cdot}4458192223320340849$. Jonny suggests readers might find it fun to tackle these numbers with Wolfram Alpha.

On a lighter note, perhaps it is time for a partial resurrection of our annual puzzle. The idea is to see how far you can get expressing numbers using the digits of the year once and once only in order, using the operations +, - x, divide, factorial, square root, and concatenation. Thus, to get you started, 1 = – 2 + 0 – 1 + 4.