Maths is Beautiful

Mathematics possesses a beauty cold and austere, sublimely pure, and capable of a stern perfection such as only the greatest art can show.

Bertrand Russell (1872-1970)

No subject creates such an extreme set of emotions in its students as mathematics. Some endure a phobia so distressing when faced with anything labelled as maths that they freeze, failing to trust in what ability they actually have and counting the days until they can drop the subject. Others find maths so fulfilling, and spend so much time in that mysterious world, that observers accuse them of neglecting the roundedness of their personalities. These feelings pursue us into adult life; a friend experiences palpitations when her Health and Social syllabus asks her to teach a mathematics element, while when another friend wins a day to himself (a birthday, perhaps), he always says, ‘I’ll take it doing maths.’

This second friend is in danger of being misunderstood. He does not want to ‘do sums’ all day; he will not be ploughing through past exam papers. The truth? He’s profoundly aware of the transcendent beauty that lies behind the best mathematics. A day spent pursuing that beauty, in tackling some delicious problem, or in reflecting on the extraordinary mental structures that mathematicians down the ages have conjured up as discoveries, is in his view a day well spent. Such a day of wonders, he would argue, tends to leave your character more rounded and not less.

Some may doubt him. ‘What does this mathematical beauty look like?’ I hear you ask. Let me pick an example. If you and a colleague have a moment, you might try this quick game. Imagine that you have nine cards bearing the digits 1 to 9 lying on the table in front of you.

1	2	3	4	5	6	7	8	9

You take it in turns to pick a card. The winner is the first person to have exactly three cards in their hand that add up to 15. A sample game might go like this:

Choice	1	2	3	4
A	6	5	8	2
B	3	4	1	

Player B is forced to pick 4 as their second choice to block A from getting 6-5-4, but then A is forced in turn to block Player B by picking 8. Player B is then in an impossible dilemma; he has to pick both 1 and 2 to block A, and so he loses – B picks 1, and Player A ends with the winning combination 2-5-8 in their hand.

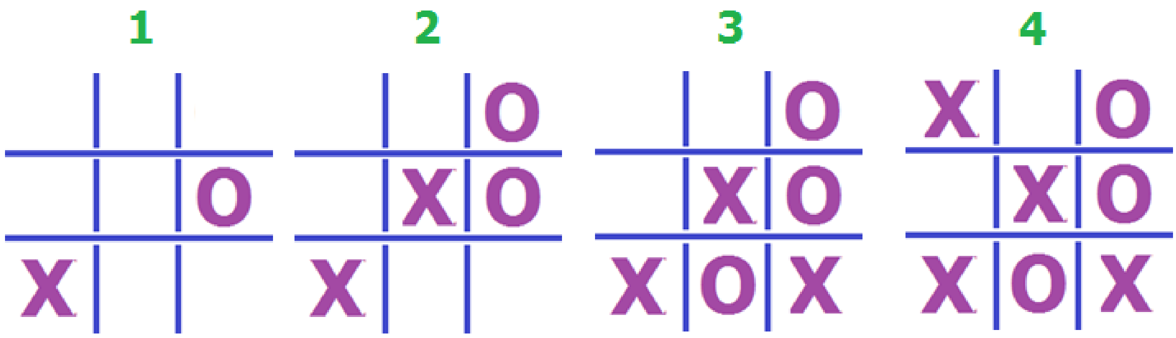
A bagatelle to while away a minute, you might think – but a mathematician drawn into this might find herself asking questions. Is it better to go first or second? Is it guaranteed that with best strategy the game will end in a win for one side or the other, or is a draw possible? What is this best strategy? What if the game has more cards, and what if the target number 15 can vary? How about a three-player version? Generalising, extending, varying, these are all things that a mathematician does. 'How can I look at this in a different, more helpful way?' Perhaps, even, in a more beautiful way...

Suppose our mathematician picks the cards up and rearranges them on the table. 'The numbers from 1 to 9, and the number 15,' she reflects, triggering a memory. She places the nine cards like this:

2	9	4
7	5	3
6	1	8

‘Each row adds to 15, each column adds to 15, and each of the main diagonals adds to 15. The game asks me to pick three cards that add to 15 – in other words, I am trying to get a line of three.’

Suddenly it dawns, the game is in fact - Noughts and Crosses. Suddenly all the questions about strategy are resolved, since we’ve all played Noughts and Crosses a thousand times - with best play, the game should always end in a draw, although if you are playing an inexperienced player, it does help to go first. Translating our sample game into Noughts-and-Crosses-speak helps us to appreciate Player B’s problem more keenly.



There is one check we need to make – is EVERY way of making 15 with three cards represented in our magic square? A little research shows the answer is ‘Yes’ – there are exactly eight different ways of making 15 with three different cards, and there are exactly eight different ways shown in our magic square. The analogy is perfect, and the underlying structure for these two apparently different games is seen to be identical. Mathematicians love to find similar structure in different situations – they would describe the two games as ‘isomorphic’. Spotting an isomorphism saves you work – insights concerning the first side of the isomorphism cross over into understandings concerning the second, and vice versa.

So the argument above takes an unfamiliar game and helps us realise that is in fact a familiar one. What is that enchants me about this logic? We start with something one-dimensional (a line of cards), and turn it into something two-dimensional (a magic square of cards). I feel a slight uneasiness playing the initial game (‘something tells me I have seen this before...’) which is then sweetly resolved by the magic square arrangement. There is the precision of the fact that there are exactly eight ways to get three cards to add to 15, and there are exactly eight ways to read 15 from the magic square – if you try the game with 16 cards from 1 to 16 and a 4 by 4 magic square, the isomorphism breaks down. In fact, the argument is so fragile, which is part of its beauty - change the number 15 to 14 or 16, and the logic evaporates. Now, too, I have a new implement for my mathematical toolbox – whenever I see a problem that reminds me of this one, I will always ask if this method could be used. And of course, magic squares of various sizes have a rich history of their own...

It may be that this solution to finding the game's best strategy does nothing for you. I recall a story about a woman who asked Fats Waller what jazz was. 'If you have to ask, you ain't never gonna know!' was the reply. If you are unaffected by this piece of mathematics, you might feel angry; *a lot of fuss about nothing, while true sources of beauty like poetry, art and music are ignored.* My reply would be that no mathematician claims that their appreciation of beauty trumps anybody else's. Beauty is there to be perceived by every teacher in every subject in the curriculum, whether it's a chemist contemplating the benzene ring, or a business studies teacher reflecting on the most worthwhile way to set up a company, or a P. E. teacher who has just witnessed an extraordinary piece of sport. To set up a competition between our respective beauties would be ugly in the extreme.

So for all of us the question is the same – how can I pass on beauty? I gaze out each year at my new groups, and wonder what their prior experience has been. How many will have chosen to pursue the beauty of maths beyond the syllabus in their spare time? The percentage will be small – how can we increase it?

I think back to my first glimpses of mathematical beauty, and I definitely 'caught' that love from an inspiring teacher. One day, Mr Russ came in and tried something completely different - he put a simple problem about points and lines onto the board, and I found myself gasping, 'So THIS is what maths is!' So we as teachers need to first and foremost nourish our own perceptions of our subject's beauty. We need to find snippets of time in impossibly busy lives to read around the bare necessities of what we do. We need to make sure our schemes of work contain more than simply the effective and the expedient, and we need to fight for space for the awe-inspiring, even if it doesn't nail a particular exam topic immediately. Of course, the effective and the expedient will inevitably be in there, but even apparent drudgery can be approached in a multitude of ways, some of which acknowledge beauty more than others. As the great mathematician Gauss once said, ‘You have no idea how much poetry there is in the calculation of a table of logarithms!’

The story (as told by Simon Singh) of the proof of Fermat’s Last Theorem is my favourite piece of mathematical television. There is a moment there when Andrew Wiles tries to communicate the experience of finding the final piece of the jigsaw. This gentle, brilliant man sits on film for maybe ten seconds in complete silence as he grapples with the enormity of the vision granted to him. It is as if at that point he reached the mountain top, to gaze out into a virgin valley beyond that he had somehow completely subdued through the lonely power of his fallible human mind. ‘It was so indescribably beautiful’, he says quietly, like Moses attempting to describe meeting God in the burning bush.

And in the end, it is only the spiritual that suffices to talk of any perception of beauty. I myself am a theist (I might be wrong), but there are plenty of atheists out there leading profoundly spiritual lives who I hope would agree. We live at a time when the spiritual is under threat, especially perhaps in our professional lives as teachers. Bureaucracy that is thoughtful can be beautiful too, but when it becomes an end in itself, it clubs the wondrous out of our teaching. There are few teachers who perceive the beauty in marking; increase our marking workload, and the beautiful is likely to take a hit. ‘Man cannot live by bread alone,’ Jesus taught us, but it doesn’t stop lots of people from trying, including those who prescribe us a vast administrative workload and a utilitarian syllabus.

I have three raw goals in my teaching – the shallowest is to get decent exam results. Without these, I won’t have a job, and my students will be disadvantaged, which would be unfair. Secondly, I want us all to enjoy the process of tackling maths together – the social interaction, the battling alone with homework, the overcoming of failures, the discussion, the experience of concentrating hard in a group. If I achieve goals one and two, then I keep my managers off my back, but have I even begun, really, the task of education? My third goal, the profoundest, which is really why I am a teacher at all - I want to initiate my classes into the human conversation that is mathematics, and that includes the appreciation of mathematical beauty. I want us to be able to look back at our hours in my classroom together as a time of wonder – not non-stop wonder, for that would be too much to bear, but I want us to lose any fear of mathematics that we might have had, even in the face of real difficulty, and I want us to learn that everyone on this earth is a mathematician, even if they don't know it yet.

Do I manage this? My colleagues will read this and smile; I think on my classroom as it is now, and I know my ideals do not make it into practice just yet. I still make the mistake of telling my students in advance, 'Now here's a beautiful piece of maths coming,' only to be met afterwards by blank faces; 'was that it?' Some of my students have learnt that I look for elegance in mathematics; they will mischievously raise their hand after seeing a smart solution and ask, 'Would you say that that was 'elegant', Jonny?' But then, I've only been teaching, and trying to get mathematical beauty across, for 25 years - ask me how I'm doing in another 25 years time.

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