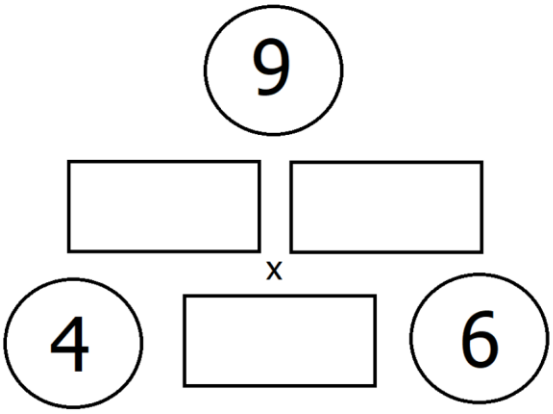
6
D1GITI53RS
2 4

Digitisers

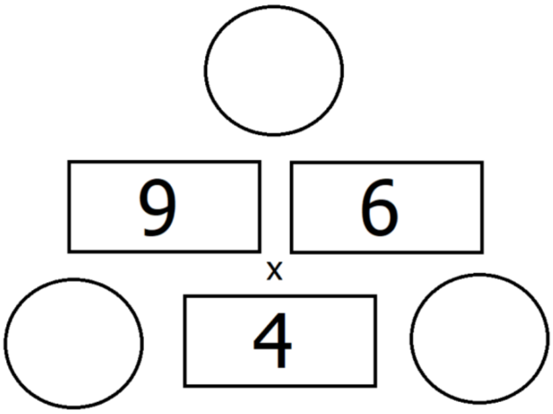
I was lucky enough to work for nearly ten years under an excellent Head of Maths called Pete. He was a fine mathematician, his students loved him and he worked his socks off for the department. Pete was, however (and I hope he would agree with this), a Hafti, that's 'a Hard Friend to Impress'. So when one day he asked me for a resource on vectors to present to his Year 12 group, I chose carefully. And when Pete came back saying that his group had loved working on it, my maths resource antennae sprang into action. If Pete said it had worked well, it must have done.

What I'd given Pete was a digitiser. I should say the concept is not remotely original (although maybe the name is); indeed, you can see tasks of this type on Twitter on any day of the week. The idea for my version is to write down a true mathematical statement that involves numbers. You then rub out the digits from 1 to 6 once each, and the task becomes to put them back in again truthfully. If the task has been properly constructed, there should be a unique way to do this, and so completing the puzzle involves not only finding a solution, but showing that it is the only solution.

Pete's feedback left me feeling that I had a format that deserved to be explored. I think of the arithmagon, an arrangement of circles and rectangles that can generate endless fruitful maths explorations. In the arithmagons below, the numbers in two circles multiply to give the number in the rectangle that separates them.



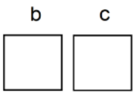
What is in the rectangles?



What is in the circles?

There is here the idea of first 'doing', but then 'undoing'. If one of our students can both do and undo when working with a particular concept, we can be sure that mastery is not far behind. If an arithmagon can be described as a format, then it's a valuable one. Maybe the digitiser could be valuable in the same kind of way.

So the best thing is probably to just throw a few digitisers your way, and see what you make of them. The ones I've written have been late GCSE/early A Level in terms of syllabus, although of course a digitiser could be written for any level. The solutions for the first two have been included so that you can be sure you see what is going on. If



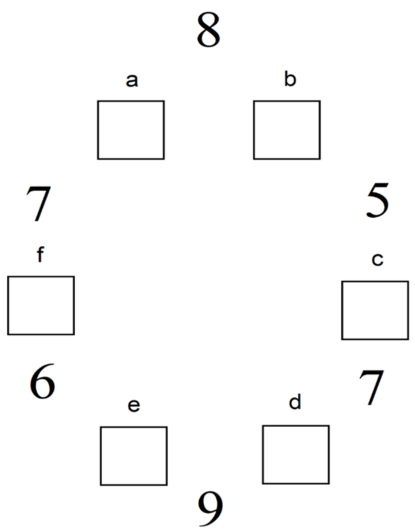
appears somewhere, this is the two digit number with first digit b and second digit c. Your final answer each time should be the six-digit number abcdef. I offer these tasks at three levels of difficulty – here is a gentle starter, rather akin to an arithmagon.

Digitiser A

Difficulty: *

Topic: arithmetic

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)



Two neighbouring squares add to the numeral between them.

Digitiser A Solution

What can b be? Not 1, since then a = 7.

b could be 2 or 3, but not 4, since then a would be 4.

b can’t be 5 or 6, since then c is 0 or -1.

So b is 2 or 3, and c is 3 or 2, and 2 and 3 are taken.

So d is 4 or 5, and e is 5 or 4.

Thus 2, 3, 4,and 5 have all been used, and f and a must be 1 and 6.

But f can’t be 6, for then e would be 0,

so f = 1, e = 5, d = 4, c = 3, b = 2, and a = 6.

Everything checks out, and the required answer is 623451.

Digitiser B

Difficulty: **

Topic: completing the square

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

$$x^2 + \overset{a}{\square}\,x + \overset{b}{\square}\overset{c}{\square}$$

$$= (x + \overset{d}{\square})^{\overset{e}{\square}} + \overset{f}{\square}$$

Digitiser B Solution

To get the x^2 term on the right hand side, e = 2.

a = 2d, so a = 6, d = 3 (all other options involve the 2).

Thus f + 9 = 10b + c, and b cannot be 4 or 5 here, so b = 1.

Now we must have f = 5, c = 4 (the other way around won't work).

Answer; 614325

Digitiser C

Difficulty: ***

Topic: trig functions

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

$$A = \overset{a}{\square}\overset{b}{\square} \qquad B = \overset{c}{\square}\overset{d}{\square}$$

$$C = \overset{e}{\square}\overset{f}{\square}4$$

$$0 < A < B < C < 180$$

$$\cos A^o = \sin B^o = \sin C^o$$

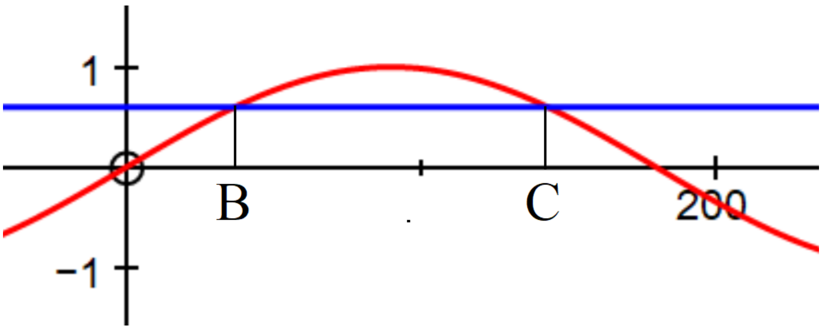
What do I like about these puzzles? If you are looking for a consolidation starter, these should take the perfect length of time. They're a lot of fun to write, which is a good sign. Indeed, asking students to construct their own is a great extension. They pack a lot of maths into a very small space. Targeting a particular part of the syllabus that you want to revise is easy; just write your initial statement accordingly. The logic involved is worthwhile, and digitisers support ideas of proof well. I would say, however, that there is a balance to be struck here; if the reflecting-on-maths-theory element to your digitiser takes 5 minutes, while the sorting-out-the-logic takes half an hour, the balance of your task may not be quite right.

I've combined my first fifty attempts at a digitiser into an ebook called 'Digitisers' that is available for free on my website (just google 'Jonny Griffiths'). I hope this might become available also somewhere on the ATM website soon. My further hope is that my efforts will be competing with plenty of others before too long…

Jonny Griffiths, hello@jonny-griffiths.net, Feb 2022

Digitiser C Solution

Clearly e = 1.



The graph of y = sin x above tells us that

if sin B° = sin C° and 0 < B < C < 180,

then C = 180 – B, or B + C = 180. Thus d = 6.

Also sin B° = cos (90° – B) = cos A°.

cos P° = cos Q° means that P = Q + 360n° or P = -Q + 360n°.

So A = 90° - B + 360n° or A = B – 90° + 360n°.

90 = B – A is impossible, since B – A < 54 - 12.

That means that A + B = 90, so b is 4.

Since A < B, B > 45.

B = 46 gives A = 44, impossible.

B = 56 gives A = 34, C = 124.

Thus a = 3, b = 4, c = 5, d = 6, e = 1, f = 2.

Answer; 345612