The Sifter

Jonny Griffiths, Jan 2013

Consider the curve $y = ax^2 + bx + c$.

Where does it cross the x-axis? How many roots do we have?

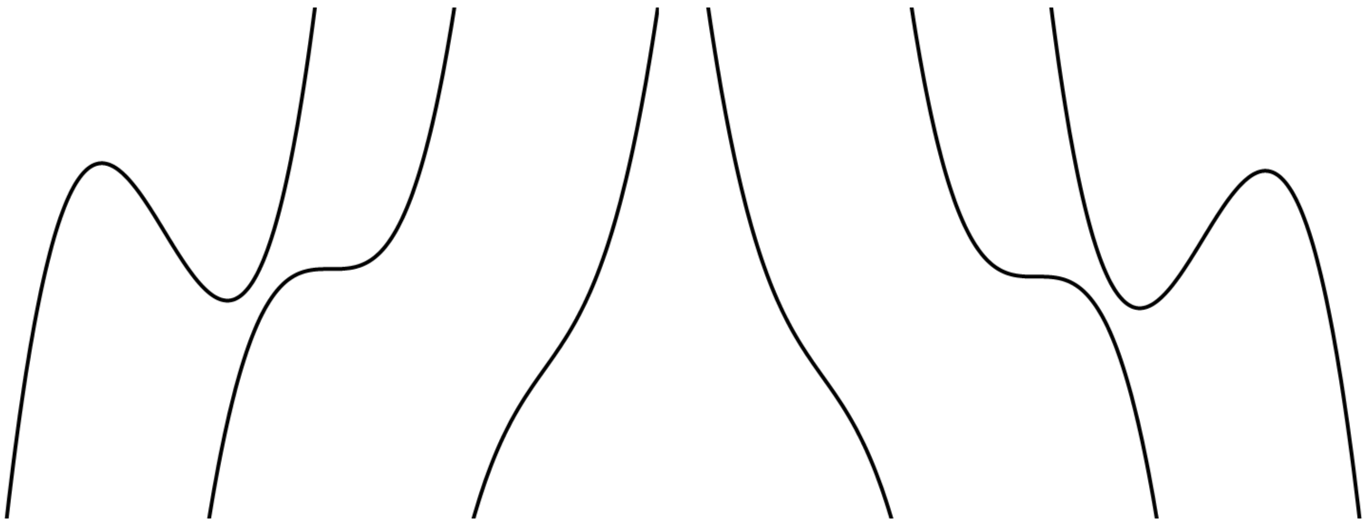
We know the *discriminant* here is $\mathbf{b^2 - 4ac}$.

This discriminates between the three situations

1. The equation has two distinct roots ($\mathbf{b^2 - 4ac} > 0$)
2. The equation has one repeated root ($\mathbf{b^2 - 4ac} = 0$)
3. The equation has no real roots ($\mathbf{b^2 - 4ac} < 0$)

Now think about $\mathbf{b^2 - 3ac}$. What could this represent?

Let's consider cubic curves; what shapes can they have?



They can have two turning points, or a point of inflection,

or no stationary points at all.

Consider the curve $y = ax^3 + bx^2 + cx + d$.

How many turning points does this have?

y' = $3ax^2 + 2bx + c = 0$ when

$$x = \frac{-2b \pm \sqrt{4b^2 - 12ac}}{6a} = \frac{-b}{3a} \pm \frac{\sqrt{b^2 - 3ac}}{3a}.$$

So if $\mathbf{b^2 - 3ac} > 0$, we have two distinct turning points.

If $\mathbf{b^2 - 3ac = 0}$, we have one repeated stationary point,

that is, a point of inflection,

and if $\mathbf{b^2 - 3ac} < 0$, we have no real stationary points.

Note that if $\mathbf{b^2 = 3ac}$, then $y' = 3a(x + \frac{b}{3a})^2$

We might call $\mathbf{b^2 - 3ac}$ 'the sifter'.

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