<u>Theory into Practice goes Exactly</u>

Last Christmas my rather rusty violin skills were pressed into service for the College production of *Animal Farm*. The ending to Orwell's dystopian masterwork was as bleak as it was possible to make it, leaving the audience suitably challenged. As I played through five performances, I could not help but reflect on the mightily dysfunctional relationship between theoreticians and practitioners in the piece. Does power always tend to corrupt, as it did with the pigs, and does power always reside with the theoreticians? Could the sheep and goats and horses have been less submissive as practitioners? Maybe the play had some uncomfortable resonances for my life as a mathematics teacher.

So how should theorists and practitioners get along? (Of course, we are all likely to be a bit of both.) If things go well, there will be a harmony present, each realising their dependence on the other. If the relationship breaks down, then the practitioner will weary at the words of the theorist, that can seem so remote from the daily hurly burly of the job itself, while the theorist may find the practitioner's attempts to engage in theorising rather gauche and embarrassing. Theorists coin words, and if that activity becomes an end in itself, these neologisms become the chatter of a detached elite. Yet new words induce new thoughts in those who hear them, and how can you consciously improve your practice without thinking new thoughts?

At the time I was teaching a GCSE re-sit group, and wondering how to get them to re-address that old chestnut of 'percentages'. I took the following question:

**A coat costing £130 is reduced by 20%,
and then by a further £15. How much does it now cost?**

Was this a chance to apply some of the theory of mathematics education in my daily practice? ***Dimensions-of-possible-variation*** is a phrase that now occupies a safe place in the literature.

Asking yourself what could be changed [in the task], while using the same approach or technique, opens up ***dimensions-of-possible-variation****. A set of exercises forming a sequence of tasks for learners to work through piecemeal [is transformed] into a domain of generality – a technique in the full sense of the word. (1)*

What can we vary in my simple percentage problem? The number 130 for sure. And the numbers 20 and 15? Yes, but I decided to commit myself to these two figures. Maybe, however, I could alter the order in which they appear. We also have a percentage reduction and a fixed reduction: could I not vary the order here too? My problem became the following set of eight problems:

A coat's starting price is £130. Which is the best deal?

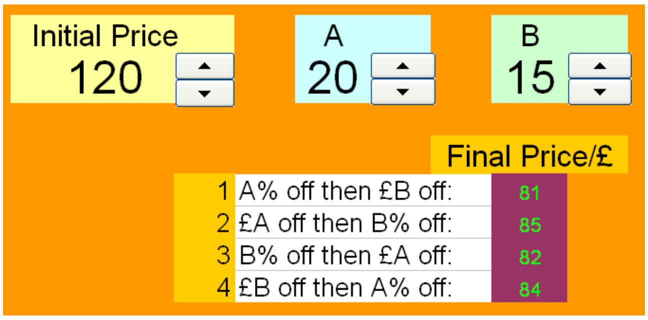
1. **Sale price 20% off! Now reduced by another £15!**
2. **Sale price £20 off! Now reduced by another 15%!**
3. **Sale price 15% off! Now reduced by another £20!**
4. **Sale price £15 off! Now reduced by another 20%!**

Which is the best deal if the starting price for the coat is £80?

I placed the problem onto the OHP, and watched my students carefully to see what I call 'The Fuzz' descend. Eyes around the room narrowed, as they tried the problem for size. To experience The Fuzz is to say, "I haven't quite got a handle on this, but I'd like to have a handle on it." If The Fuzz turns out to be too painful, then my students will throw in the towel, saying, "This is beyond me." If The Fuzz proves negligible, then the problem will appear routine and pedestrian, and I might as well have used a textbook exercise. But if the problem is pitched correctly, then curiosity is awakened, and my students will be drawn into the problem. Engagement takes place - they will have a desire to resolve The Fuzz for themselves.

I often get my students to vote in situations like this. Having to nail one's colours to the mast early on in a problem, to make a visible commitment, is important. I recall some research showing that those who change their minds least in a meeting are those who say nothing. No one was prepared to vote for Deal 1, so Katie did - "I'm a bit of a rebel," she said. Deals 2 to 4 attracted votes in equal measure, before everyone settled down to tackle the maths in earnest. The winner after lots of feverish activity? Deal 1, leaving Katie with the Intuition Prize of the day.

What happens if we change the starting price of the coat to £80? To our surprise, we found that the maths could now be done in our heads, even with our eyes shut. The best offer this time turned out to be Deal 3. Further investigation was greatly aided by the following Excel spreadsheet:



This can be downloaded at http://www.s253053503.websitehome.co.uk/percentage-reduction.xls

My GCSE resit students (rightly) complain that simultaneous equations are highly unlikely to be of any use in real life to any of them. Yet percentages are vital for everyone. The difference between the offers here is only a few pounds, but if you buy a house rather than a coat, you can multiply that by a thousand.

So the theory genuinely helped my practice. Considering ‘dimensions-of-possible-variation’ achieved exactly what the quotation above promised - a potentially dull and routine problem was transformed into one of genuine interest, where my students developed a technique in a spirit of curiosity.

I have another activity which requires mixing up the order of doing things to practice dividing and multiplying by 100. Give your students the following four cards:

+ 100	− 100	x 100	÷ 100

The task is to pick a number, then to apply these four cards in some order. How many orders are there? Students quickly realise that placing the + and – cards together, or the x and ÷ cards together, means that everything cancels out. So there are eight orders worth looking at:

1. + x - ÷	5. - x + ÷
2. + ÷ - x	6. - ÷ + x
3. x + ÷ -	7. ÷ + x -
4. x - ÷ +	8. ÷ - x +

How many different numbers can you make? It turns out that whichever number is chosen first, exactly four different numbers will be created, and the order (smallest to largest) will always be as follows:

2 = 8, 3 = 5, 1 = 4, 6 = 7

A curious and worrying thought: “Changing the starting price of the coat altered the order of offers from worst to best. Yet changing the starting number in this activity has no effect on the final order. Why should this be?” Now it is my turn for The Fuzz to descend…

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1. *Fundamental Constructs in Mathematics Education*, edited by John Mason and Sue Johnston-Wilder, Routledge 2004, pg 58.