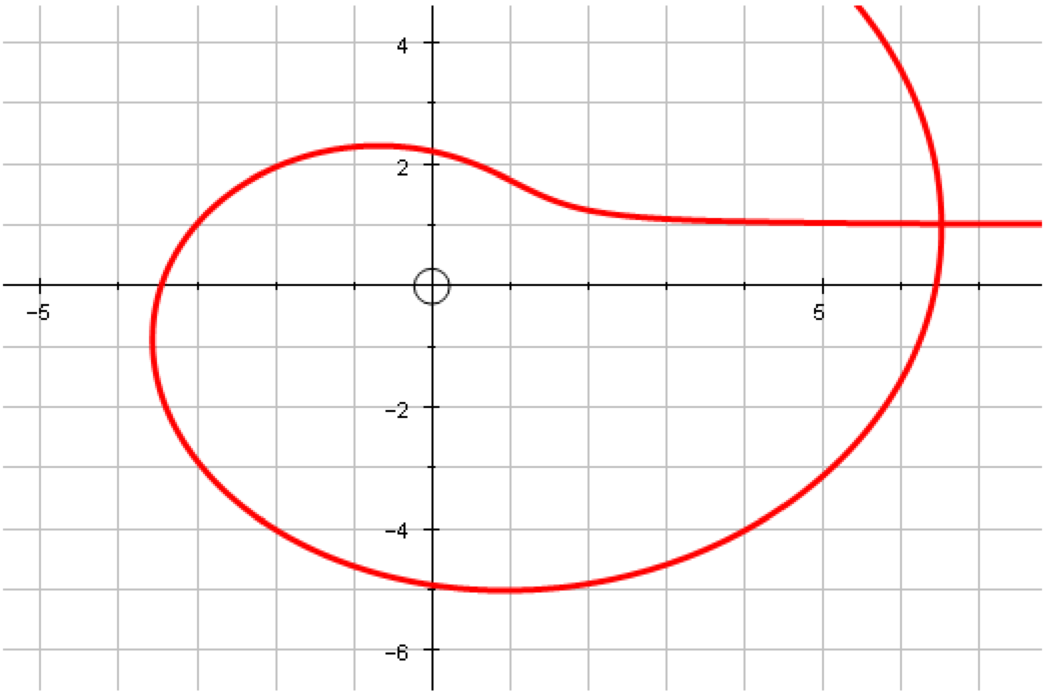
The curve $r = \theta + \dfrac{1}{\theta}$ for $\theta > 0$ encloses a central area – what is this area in terms of π?

<u>Solution</u>

The curve looks like this at the origin:



Let the coordinates of the point of intersection be (a, α).

Then $\alpha + \frac{1}{\alpha} = (2\pi + \alpha) + \frac{1}{2\pi + \alpha}$

Multiplying by $\alpha(2\pi + \alpha)$ we get;

$\alpha^2 + 2\pi\alpha - 1 = 0$

So $\alpha = \sqrt{(\pi^2+1)} - \pi$

So area = $\int_{\sqrt{\pi^2+1}-\pi}^{\sqrt{\pi^2+1}+\pi} \frac{1}{2}(\theta + \frac{1}{\theta})^2 \, d\alpha$

$= \left[\frac{\theta^3}{6} + \theta - \frac{1}{2\theta}\right]_{\sqrt{\pi^2+1}-\pi}^{\sqrt{\pi^2+1}+\pi}$

= (after a lot of neat simplification!) $\frac{4\pi^3}{3} + 4\pi$.

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