Risp 23: Teacher Notes

Suggested Use: to consolidate/revise degrees/radians and general trigonometric equations

There are many ways to present this activity. You could gently pick on the member of your class who has the reputation for being most scatty. The truth is we have all initiated calculations that have failed because we have worked with a calculator in radians rather than degrees or vice versa. This risp imagines the happy coincidence when we get away with it.

So we need to solve: $\sin x^{o} = \sin x^{c}$.

We know that $1^{c} = \dfrac{180^{o}}{\pi}$, so we seek to solve $\sin x^{o} = \sin \dfrac{180x^{o}}{\pi}$.

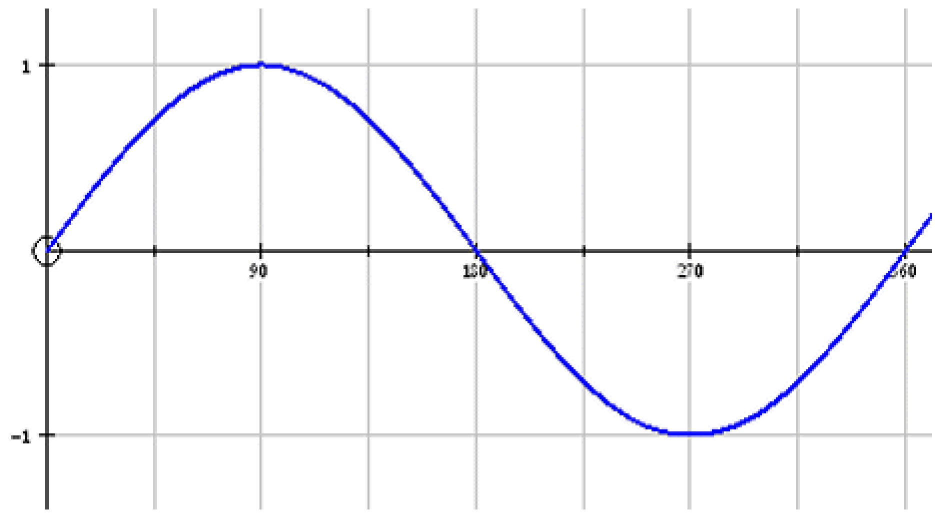
The general solution is $x = \dfrac{180x}{\pi} + 360n$, or $x = 180 - \dfrac{180x}{\pi} + 360\,n$.

These solve to give $x = \dfrac{360n\pi}{\pi - 180}$ [Formula 1]

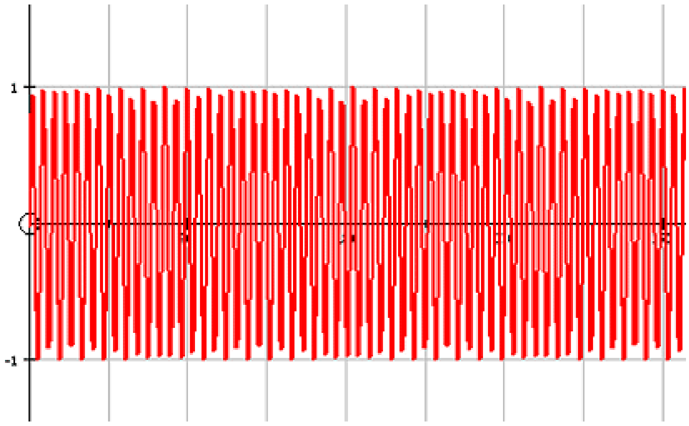
or $\dfrac{(180 + 360n)\pi}{\pi + 180}$ [Formula 2].

Now it is a question of trying out values for n to see if we can get a value for x between 45 and 50. This happens only if we put n = 7 in Formula 2, giving $x = 46.31553129...^{o/c}$.

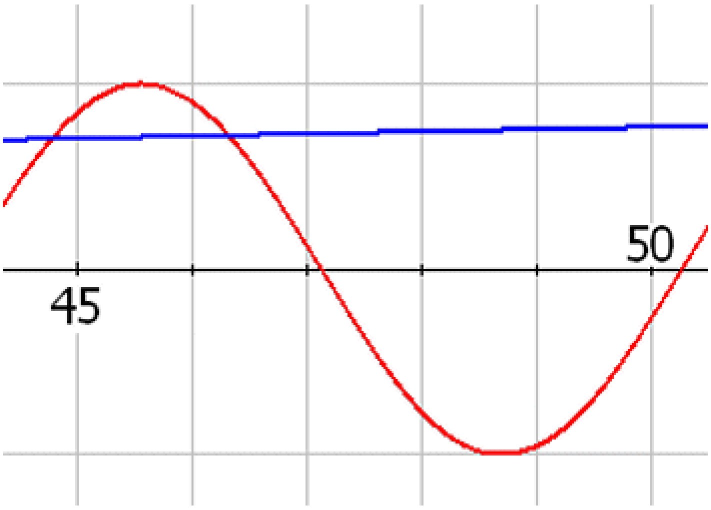
It is nice to tackle this graphically. The graph of y = sin x looks like this:



But change degrees to radians here and we get this:



Plotting these together gives lots of crossing points, but only one in [45, 50].



What if we insist also that $\cos x^{o} = \cos x^{c}$?

This gives $x = \dfrac{180x}{\pi} + 360n$, or $-\dfrac{180x}{\pi} + 360n$. These solve to give:

$$x = \frac{360n\pi}{\pi - 180} \text{ [Formula 1] or } \frac{360n\pi}{\pi + 180} \text{ [Formula 3].}$$

So Formula 1 will always give a value x so that $\sin x^{o} = \sin x^{c}$ and $\cos x^{o} = \cos x^{c}$.

Which of these values is closest to 46.31553129...?

n = —7 gives x = 44.76356881... as shown on the graph above.